



Simulation of an Active Suspension Control for Frequency Response from Road Vibrations Using H-Infinity Synthesis and Validation for Robust Operation Using M-Synthesis

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Abstract

In a general suspension system ride comfort and suspension performance are inversely related. Whenever the vibrations are damped in a periodic and slow manner, the ride comfort improves but the suspension performance degrades. This project presents a frequency-based controller for active suspension using the H-infinity algorithm. The implementation includes performing the analysis using a general quarter car model. This is done to reduce the complexity of the model and at the same time helps to carry out the analysis without any compromise. In the implementation, the quarter car model is converted into a state-space model to make further analysis easier. The next part includes the development of a robust controller so that the system response is acceptable even in the case of uncertain input. In the model developed, the output states to be monitored are taken as suspension deflection and body acceleration. The input state to the controller is taken as the actuator force input. Intent of the model development is to mitigate the general trade-off between ride comfort and suspension performance.

Keywords: Active suspension, quarter car, hydraulic actuator, state-space, weighting function

1. Introduction

Active suspension system represents the highest level of performance in the suspension types. There is a general trade-off between ride comfort and suspension performance in every suspension system. As the designer goes on improving the ride comfort, handling degrades. This happens because system performance is defined by the vibration damping characteristics of the system. The faster the damping the better the system performance. On the other hand, ride comfort is measured on the basis of how smoothly the system damps out the vibrations. In the project, the main objective kept in mind is to design such a suspension system which mitigates this problem to a considerable extent. Suspension deflection and body acceleration are the two main parameters to be analyzed.

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In the model developed, the output states to be monitored are taken as suspension deflection and body acceleration within the limits -0.315m/s^2 to $+0.315\text{m/s}^2$ for comfort and -2m/s^2 to $+2\text{m/s}^2$ for handling as per ISO 2631 standard used by Fabian Leon Vargas et al [9]. The input state to the controller is taken as the actuator force input with a minimum force capacity of 2.08 kN for lifting the sprung mass. The road profile used for testing the system was developed by Mohammad M ElMadany et al [7].

1.1. Aim

To simulate an H-infinity controller for a general active suspension system using frequency response to road excitations to mitigate the general trade-off between ride comfort and performance and making the system robust using Mu synthesis.

1.2. Objectives

- To convert a quarter car suspension model into state-space model
- To design a controller for the model using frequency analysis of road excitations
- To define and test the system for external disturbances (road excitations)
- To calculate suspension deflection and body acceleration as a function of frequency to gain insight into the performance of the system
- To make the system performance robust which means that the body acceleration values should lie within the range of -0.315 m/s^2 to $+0.315\text{ m/s}^2$ for the comfort mode -2 m/s^2 to $+2\text{ m/s}^2$ for the handling mode and the system be stable in the frequency range even when the input (the road excitations may vary till 100% from the given amplitude limit) by employing the μ synthesis
- To ensure a proper balance between handling where it refers to damping the vibrations with stability

2. Literature review

2.1. Quarter car model

Quarter car model is the reduced scale representation of the whole car. It is used because it reduces the complexity confronted while using a whole car model as well as reduces the computation time and the results can be easily scaled to a full car model. In the model, a quarter car model employed by Wiechao Sun et al [11] is used to carry out further analysis. Newton's second law is applied on the model to represent equations of motion.

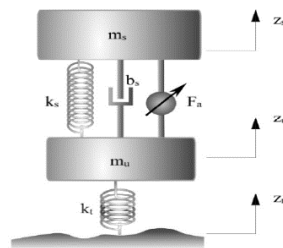


Figure 1: Quarter car mode representation

➤ Parameters:

1. Sprung mass ($m_s = 320\text{kg}$)

2. Un-sprung mass ($\mu = 40\text{kg}$)
3. Spring constants of the suspension ($k_s = 18000\text{N/m}$)
4. Spring constants of the tire ($k_t = 200000\text{N/m}$)
5. Damping coefficient of the damper ($b_s = 1000\text{Ns/m}$)

The above model can be broken down according to Newton's laws of motion as follows:

$$\mu \ddot{z}_u + c_s(\dot{z}_u - \dot{z}_s) + k_t(z_u - z_q) + k_s(z_u - z_s) + c_t(\dot{z}_u - \dot{z}_q) = 0$$

$$m_s \ddot{z}_s + c_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = 0$$

2.2. Road profile

2.2.1. Assumptions

- i) The road profile is assumed to be sinusoidal with frequency fixed at 0.314 rad/s
- ii) The maximum amplitude of the profile is fixed at 0.0025m

In the model, the road profile taken is sinusoidal and based on power spectral density. Power spectral density (PSD) is a measure of the signal's power content versus frequency and is used to characterize broadband signals. PSD shows the variation in the amplitudes over the frequency range as done by Verros, G. Papadimitrou et al [8]

$$x_g(t) = \sum_{n=1}^N s_n \sin(n\omega_0 t + \phi_n). \quad (2)$$

where x_g is the value of the amplitude at a given instant of time and s_n is the maximum amplitude of the road profile given as:

$$s_n = \sqrt{2S_g(n\Delta\Omega)\Delta\Omega} \quad (3)$$

$$\Delta\Omega = 2\pi/L \quad (4)$$

where L (1000 m) is the length of the road and Ω is the spatial frequency and S_g is the roughness of the road and the value of Ω comes out to be 0.00628 m^{-1}

The value of s_n comes out to be 0.0025 m for bad roads and 0.00063 m for good roads. For good roads:

$S_g = 16 \times 10^{-6} \text{ m}^2 \text{ cycle}^{-1} \text{ m}^{-1}$ (taken from ISO 2631 standard)

For bad roads: $S_g = 256 \times 10^{-6} \text{ m}^2 \text{ cycle}^{-1} \text{ m}^{-1}$

2.3. H-infinity method

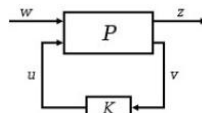


Figure 2: H-infinity parameters

As can be seen in fig 2, an H-infinity controller consists of the main system or plant (P) and the feedback

controller (K) as suggested by Miaomiao MA et al [14]. K maps measured quantity ‘y’ onto control input ‘u’ as $u=Ky$, where y are the measured output quantities (in this case body acceleration and suspension deflection). P is the plant matrix.

$$\begin{bmatrix} z \\ v \end{bmatrix} = \mathbf{P}(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \quad (5)$$

w is the input (road excitations in this case) and u is the controlled input (actuator force), z is the error signal which needs to be minimized, v represents the measured variables.

The basic objectives in minimizing the norm:

$$\bar{z}(s)\omega = [D + C(sI - A)]B \quad (6)$$

2.4. μ -synthesis

Robust system implies a system which can give results within limits of permissibility even in the case of uncertain input, for the given case the robustness criteria should be satisfied with the input varying as 30 % t ‘ μ ’. The expression of μ is given as:

$$\mu = \frac{1}{\det(I - \Delta M)} \quad (7)$$

M is obtained as follows:

$$M = P + (I - PK)^{-1}P \quad (8)$$

where P is the plant function given by:

$$P = D + (sI - A)B \quad (9)$$

3. Method of Analysis

- 1) Design a basic quarter car model representing a suspension model
- 2) Setup the values for road disturbance to test the system response
- 3) Finding the values of undesirable frequencies – tire hop and rattle space frequency
- 4) Design of a force actuator model (representing the actuator of the active suspension)
- 5) Allocate weights to the control effort, suspension deflection, body acceleration so as to minimize the H-infinity norm from disturbance inputs
- 6) Design of a feedback controller (nominal H-infinity controller)
- 7) Simulate suspension deflection and body acceleration as a function of time first using the regular H-infinity controller so as to see whether the parameter response is within the specified limits
- 8) Test the system for robustness using μ -synthesis

4. Results and discussion

4.1 Rattle-space and tire hop frequencies

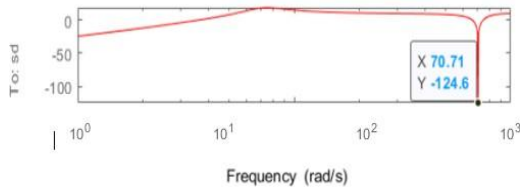


Figure 3: Magnitude plot of the transmissibility of road excitation to suspension

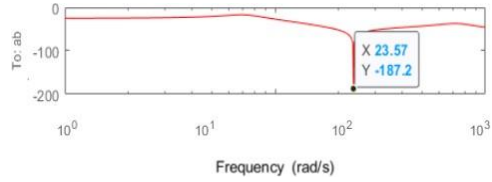


Figure 4: Magnitude plot of the road excitation to tire

Two sudden undesirable changes can be seen corresponding to rattle space (70.71 rad/s) and tire hop frequency (23.57 rad/s) respectively.

4.2 Results for regular H-infinity model

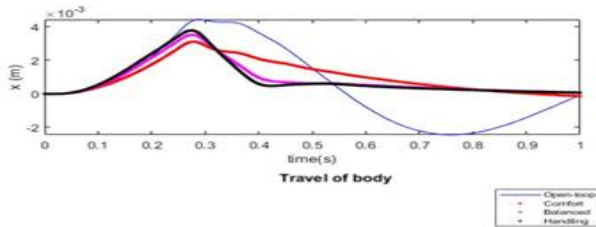


Figure 5: Suspension travel (m) vs time (s)

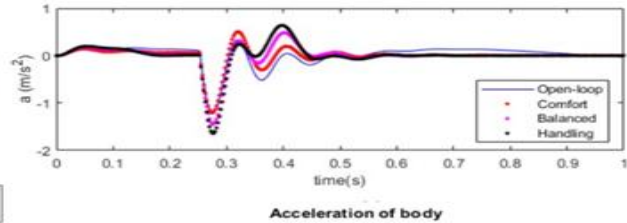


Figure 6: Body acceleration (m/s²) vs time (s)

From fig 5 and 6, it can be inferred that the graph for the handling mode settles the fastest with a maximum body travel of 0.0037 m and within the acceleration range (0.64 m/s² to -1.63 m/s²) which is the highest acceleration range. As opposite to this case, the graph for the comfort mode case follows the acceleration range (0.30 m/s² to -1.1 m/s²), for the balanced mode the acceleration lies between -1.5 m/s² to 0.42 m/s².

4.3 Validation with standard results

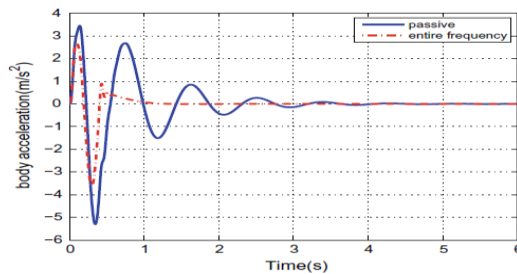


Figure 7: Acceleration (m/s²) vs time (s) graph for entire frequency H-infinity control as obtained in the research

The result comparison between the two shows a good agreement with a maximum difference of 7.1%.

4.4 Results using robustmodel

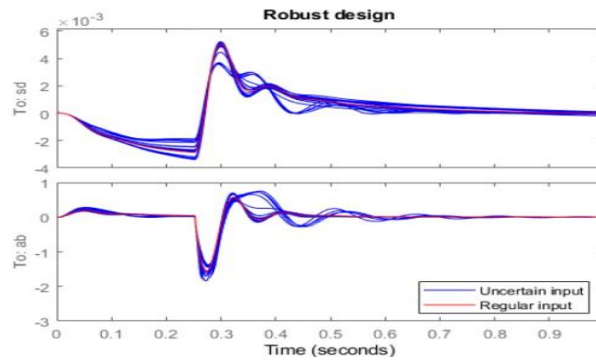


Figure 8: Output response of the robust controller for normal and uncertain input

The number of samples for the uncertain input are kept the same as 1000 in number. The peak μ value is obtained as 0.85.

5. Conclusion

In reality the suspension controller modelled would work in the normal frequency range with the exceptions of higher frequencies as can be inferred from Bode plots. The controller modes can be implemented in drive control as per the requirement of the driver. If passenger comfort is the requirement, comfort mode will be selected since it is linked with the least body acceleration and in bad road conditions handling mode will be preferred. The lower the norm, the better it will be from an economical point of view since it will lower the actuator force requirement. So, in short, to reduce the cost of the system the comfort mode would be the best choice. Increasing the uncertainty range response of the system will have a direct effect on the actuator cost and thus on the overall system cost. Again, the controller implemented is a hydraulic one which the designer can change.

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