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Crack Propagation in the Spur Gear of Tubler Gear Mechanism of Lathe Machine

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Abstract

An estimation of the spur gear service life and crack path analysisis proposed in this paper. I_p theory using numerical solution is used to find the fatigue crack propagation rate, service life, and crack path due to bending fatigue. A spur gear of tumbler gear mechanism of lathe machine,is implemented for this analysis. An experimental test on gear specimens was carried out to analyze the crack propagation due to fatigue. Besides, service life estimated by the theory was compared with the most common Paris law. Hence further,extended finite element approach (Ansys)is implemented for the simulation of gear crack propagation trajectory. The simulation results are verified with experimental test.The service life and crack trajectory of the gear is obtained with 'I_p' theory and found in good agreement with experiment.In overall, this study signifies the gear failure prediction methodology during crack propagationin respect of life cycles and trajectory path.

Keywords: Spur gear,fatigue crack growth,Bendingfatigue,Tumbler gear mechanism; Numerical simulation

1. Introduction

The tumble gear mechanism of the lathe machine is used totransmit the power and motion to the lead screw.The lead screw mechanismdeals with automatic movementof the carriage for thread cutting, boring andturningoperations, etc.The tumbler gear mechanism with gear numbers is shown in Figure1.In this application, it has been noted that, Gear-2 of the tumbler gear mechanismfrequently fails due to the consequence of crack propagation located at the gear root. This failurecan becatastrophic and should be prevented. Ifthe gear fails catastrophically, then there can be fatal accidents and financial loss to the industry or users.Prior information is necessary forservice life estimationof the Gear-2 so that it could be easily replaced before the final life cycle.It is imperative to design the gears for longer life.

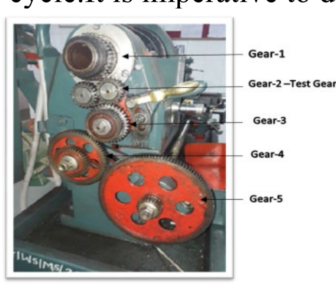

Figure 1: Tumbler gear mechanism of the Lathe machine

The gears are generally failsbyparticularreasonsof the crackgeneration due to impulse tangential load. More care must be taken whiledesigning the gear so that the gear can work for

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specific life cycles, and the catastrophic tooth failure can be avoided. For accurate determination of the crack propagation life, perfect understanding about crack growth and stress intensity factor (SIF) is prerequisite.

TheGear-1 is fixed on the lathe spindle. The lead screw is connected to input shaft through Gear train-(1-5).The Gear 2is fitted on the motor shaft which is used for the fatigue crack growth test because it isfrequently failing. An electric motor with threeh.p.power with a speed of 1440 revolution per minuteis used to drive the lathe spindle with chuck.

It is essential to study the fracture mechanics approach in fatigue loading for spur gear to know the characteristics of crack and crack propagation.Analysis of the structural as well as rotary machine component cracks can be analysed by fracture mechanics. Crack behavior in the machine parts due to static and fatigue loading could be observed by Linear elastic fracture mechanics (LEFM). The fracture mechanics method is generally used to determine the useful life of the bodiesapplied withfatigue loading. The fatigue life of elementsaredivided into three phases: 1) The phase of crack initiation, 2) The phase of crack growth, ii) the phase of the final breakage of parts. If the crack propagation studyon the component is required in a cyclically loaded component, thenfracture mechanics should be applied to fatigue propagation life estimation.

In the past, different analytical and experimental analysescarried out to examine the gear crackpath and crack propagation lifecycle. In some succeeding readings, authors notices the definite effects or influences on the crack propagation. J. Kramberger and J. Flasker[1]were used inthe boundary element method (BEM) and LEFM for crack propagation analysis.Bertsche [3]investigated the damage accumulation method for high reliability and definite gear life.Bazic et al. [4]obtained crack initiation angles by mixed-mode SIF utilizing altered loading conditions. It was observed that the modified tangential stress (max.) criteria were a better fit for uniaxial tensile stress in parts of a ductile material. Experimental investigation and numerical simulation were used by Lewicki, and Ballarini[5]to studied the thickness of the gear rim on life cycles. Numerical simulation was carried out by applying the Fracture ANalysis Code (FRANC) computer software. The crack was fabricated with a 0.5 mm crack depth.2D stress analysis was carried to the crack initiation CT or compact tension specimen.Some authors analyzed the crack paths. Robert Errichello[7] used the general AGMA and ANSI standards to analyzethe crack path and crack propagation of the gear.Glodez and Sraml[9]applied a computational model and finite element analysis using Franc2d software to estimate the residual gear life due to cyclic bending Glodez et al. [10]estimated the service life of spur gears of hardened steel 42CrMo4 material due to cyclic bending by the computational method by usingexperimental data was used.

Mohite and Kothavale[17]used the factorial design methodology for spur gear crack propagation to get optimum parameters such as material, physical properties.The optimized parameters obtained from the survey are used in the present study.Najafian and Mozafari [18]considered the thickness of the gear rim to fix the crack propagation rate and the crack trajectory of gear. An extended FEM and LEFM were used for the crack growth study of the planetary gears.Osmond Asi [20] studied on the helical gear for fatigue failure analysis. The research work presents a failure study of a helical-gear type realized in the transmission gearbox of a bus.Paris and Erdogan [22]analyzed different crack propagation laws, and the new law was investigated.The Paris law was employed to determine the crack propagation rate and service life cycles of the components.Penhan et al. [23]studied crack propagation in the tooth root portion. A particular instrument was used for testing the gear crack initiation and growth. It concludes that the tooth is frequently failed due to the defect like mechanical treatment undercut, tracks.Ukadgaonkar[25]used I_p theory in CT specimens to decide the fatigue crack propagation rate. This rate is having a functional relationship with the difference betweenthe maximum and minimum I_p parameters.Ukadgaonkar and Awasare[26]found that distortion strain energy remained stable in the critical radius circle around crack tip.. The crack was propagated along, the determined the parameter *Ip*.It can give the magnitude and the orientation of the crack.Lu Xi [28]applied the detailed notes on the gear in the crack initiation area to find remaining stress, hardness.The crack of 0.5 mmhad been fabricated at the root of the gear.

In this analysis work, an effort is made to estimate the useful service lifecycle of the gear and fix the crack trajectory, specifically, implemented in the mechanism of the tumbler gear of the lathe machine. A test on gear carried out to analyze the crack propagation; the results were compared with the Paris law. Numerical simulation was performed to decide the SIF and model crack propagation.FEA(Ansys) was applied to analyze crack growth.Therefore, the study aims to find the remaining useful lifespan of the tumbler gear to escapefrom a catastrophic failure.

2. Material and method

2.1. Material

A spur gear test specimenwas used for the analytical and experimental studies. The spur gear material is alloy steel (SAE8620).The chemical composition, physical properties, and geometrical parametersof the material are shown in Tables 1,2, and 3, respectively. The optimized gear parameters and the material were determined by the same Authors based on the factorial design approach in a previous study [2]. The same material, geometric parameters, and mechanical properties are considered in the current work, as shown in following Table 1, and Table 2.

Table 1: Physical properties of the SAE 8620 steel

Material	Young's modulus, E (MPa)	Yield tensile strength, S_y, (MPa)	Ultimate tensile strength,S_{ut}, (MPa)	Fracture toughness K_{Ic},(MPa√m)	Poison's ratio, ν	Initial Crack length,a_i(mm)
SAE 8620	2 x 10^5	390	540	75	0.29	0.125

Table 2: Geometrical Parameters of the gear

Number of Teeth, Z	Module, m, (mm)	Pitch circle diameter, d, (mm)	Pressure angle, Ø, (degrees)	Face width, b, (mm)
28	2.5	390	20^0	12.5

2.2. Theoretical treatments

2.2.1. The $I_p$theory

Ukadgaonkar and Awasare[25]investigated *I_p theory* for fatigue fracture,a new crack initiation criterion which was based on dilatational strain energy. It is useful to identifythe crack initiation angle of the machine components applied with the dynamic or static load.The present studyis used

- Todecidethe crack growth rate and estimation of gear life under fatigue loading.
- Todetermine the crack trajectory in the tooth of the gearunder static or fatigue loading.

State of stress generated on tip of elastic body crack has been studied by the Irwin. Further, derived the expressions for $\sigma_x$and $\sigma_y$concerning crack propagation angle θ. The 2-D stresses at the crack tip are σ_x and σ_y in X and Y directions, respectively[21].

Ip theory is depending upon two stress invariants I_1 and I_2. I_1 is the first stress invariant, and I_2 is other stress invariant.The relation between the stresses at the crack tip and stress invariants (I_1) and (I_2) are given in Eq. (1) & Eq. (2) respectively.

The stress Invariant,I_1,

$$I_1 = \sigma_x + \sigma_y \tag{1}$$

The stress Invariant,I_2,

$$I_2 = \sigma_x \sigma_y - \tau_{xy}^2 \tag{2}$$

Ipis the parameter, on which Ip theory was postulated by Eq.(3),

$$I_p = I_1^2 - 2I_2 = 1/2\pi r\left(c_{11}K_I^2 + c_{22}K_{II}^2 + c_{12}K_I K_{II}\right) \tag{3}$$

Where,

$$c_{11} = 1/2\left(3 + 2cos\theta - \cos^2\theta\right), c_{22} = 1/2\left(3 - 2cos\theta + 3\cos^2\theta\right) and c_{12} = 2sin\theta\left(cos\theta - 1\right)$$

The terms KI and KII are the SIF's and $'\theta'$ is an angle of crack propagation.

$$I_p = \frac{1}{2\pi r}\left[\left(\frac{1}{2}\left(3 + 2\cos\theta cos^2\theta\right)K_I^2 + \frac{1}{2}\left(3 - 2cos\theta + 3cos^2\theta\right)K_{II}^2 + 2sin\theta\left(cos\theta - 1\right)K_I K_{II}\right]\right. \tag{4}$$

Differentiating and double differentiating Eq.(4)

$$\frac{\partial I_p}{\partial\theta} = 0 \ and \ \frac{\left(\partial^2 I_p\right)}{\partial^2\theta} < 0 \tag{5}$$

The maximum value of the θ is determined by Eq. (5).As per Ip theory, the crack can be advanced in the way of theextreme value of parameter Ip. Ip parameter is directly proportional to the dilatational strain energy density.The dilatational strain energy is the main cause of crack propagation.

For the maximum value I_p, the differentiation of the Eq. (5) in with respect to angle 'θ'', is zero.

$$\frac{\partial I_p}{\partial\theta} = 0 = [(-sin\theta + \frac{1}{2}sin\theta)K_I^2 + (sin\theta - \frac{3}{2}sin2\theta)K_{II}{}^2 + (2cos2\theta - 2cos\theta)K_I K_{II}] \tag{6}$$

Rearranging the Eq. (6),

$$\left[\left(K_I^2 - 3K_{II}^2\right)/2\right]Sin2\theta + 2K_I K_{II}cos2\theta - \left(K_I^2 - K_{II}^2\right)sin\theta - 2K_I K_{II}cos\theta = 0 \tag{7}$$

Crack propagation angleθ can be determined from 'Eq.(7)'.

Differentiating Eq.(7) with respect to θ,we get,

$$\frac{\left(\partial^2 I_p\right)}{\partial^2 \theta} < 0 \tag{8}$$

Eq.(8) must be satisfied as per guidelinesof Ip theory,.

Iptheory of fracture mechanics proposed by Ukadgaonkar and Awasare[25]and used for fatigue loading by Ukadgaonkar[26] Further, it is used to calculatethe crack growth life. This theory can beapplied for the machine components subjected to mode-I fatigue loading. It has been proposed that the crack propagationlevel was in a logarithmic relation to the range of the stress invariant ΔIp and given as;

$$\frac{da}{dN} = C\left(\Delta I_p\right)^m \tag{9}$$

Fatigue crack growth rate using Ip theory is prescribed in Eq.(9). Where dN is the no. of stress cycles and dais length of crack corresponds to dN.ΔIpis the difference of the stress invariants which can be determined from Eq. (11),(12) and (13)

$$N = \int_{a_i}^{a_f} \frac{da}{C\left(\boldsymbol{\Delta I_P}\right)^m} \tag{10}$$

The ΔI_P is a change in Ip max and Ip min , the material parameters C and m can be determined from the experiment of three point bending test on single edge notched beam using standard procedure given in ASTM E 399-80.

For fatigue loading, there are maximum and minimumstresses, namely,'σmax' and 'σmin' respectively.

Stress intensity factors are prescribed in Eq. (11),

$$K_{max} = 1.12\sigma_{max}\sqrt{\pi a}\left(MPa\sqrt{mm}\right) \text{and}\, K_{min} = 1.12\sigma_{min}\left(MPa\sqrt{mm}\right) \tag{11}$$

Stress invariant parameters of Ip can be evaluated by using the Eq. (12),

$$I_{p_{max}} = \left(K_{max}\right)^2 / \left(\pi.da\right) and \;\; I_{p_{min}} = \left(K_{min}\right)^2 / \left(\pi.da\right) \tag{12}$$

The range of the stress invariants (ΔIp) is given by,

$$\Delta I_p = \left(I_{p_{max}} - I_{p_{min}}\right) \;\; \left(MPa\sqrt{mm}\right)^2 / mm \tag{13}$$

2.1.2. Bending fatigue stress

The gear tooth is here,presumed to be a cantilever beam. The length, width, and depth of the cantilever beam are gear tooth depth, face width, and tooth thickness respectively. The gear tooth is applied with a tangential tooth load. Thetangential tooth load P_t was calculated from the power and velocity of the electric motor,used in the experimental set-up. The calculated tooth load $P_t$is 423.93 N.The basic rack tooth profile type Cis suggested in (ISO 53) standard [11] for normal working conditions. Type Cprofile was used for the manufacturing of the spur gear with hob cutter.

The dimensions of critical cross section at the root gear tooth, which was determined as per the procedure of ISO 6336-6 standard [12]The tooth dimensions at critical cross section were found from the functional relations given in the standard. S_{fn} is 5.059 mm and face width, b is 12.5 mm. Tooth depth from HPSTC to critical section, h_{fn} is 3.58 mm. Putting these values in Eqns.(14),(15) and)(16),We get;

Bending moment (BM) at the root area of the tooth,

$$M = P_t h_{fn} = 1517\; N - mm \tag{14}$$

Section Modulus at the critical section;

$$Z = \frac{b\,S_{fn}^{\ 2}}{6} = 53.336\ mm^3 \tag{15}$$

Bending Stress (σ_b) at the area of tooth root is given as;

$$\sigma_b = \frac{M}{Z} = 28.44\ N/mm^2 \tag{16}$$

This bending stress is in cyclic nature because tangential impulse load acting on gear.

2.3. Numerical simulation

2.3.1. Modelling

The SOLIDWORKS is computer-aided design (CAD) solid modellingsoftware. It runs mainly on Microsoft Windows. The design engineers can sketch out their ideas, creates models and detailed drawings with dimensions. It can generate 2D and 3D models. Spur gear solid model was created in Solid-works software. The geometric parameters of the spur gear were referred from Table 2.

The initial crack length wasassumed as 0.125mm (Glodez et al.,2002).The crack, having depth 0.125 mm and width 0.13,was performed at tooth root. The crack was inclined at 45^0 pa at the root surface arc of the tooth and parallel to tooth axis(Lewicki, and Ballarini,1997).The magnified view of the gear tooth root as shown n Figure 2 (a).Figure2(b) shows the gear tooth with a crack. The pre-meshed crack model of the gear is shown in Figure 3.

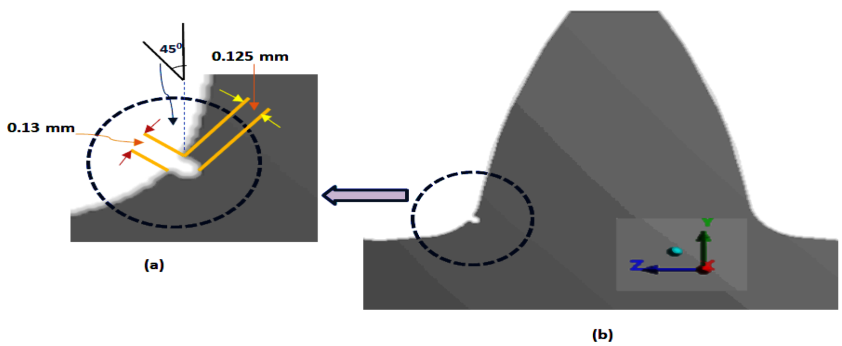
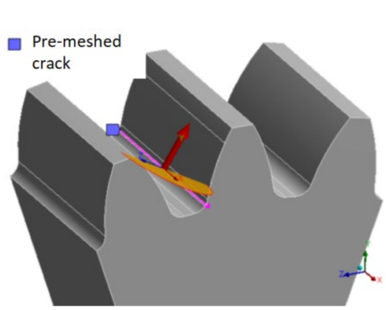


Figure 2: (a) Magnified view of the gear tooth root with crack dimension, (b) Gear tooth with a crack



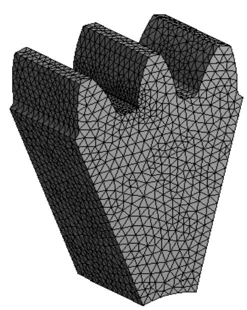


Figure 3: Pre - meshed crack in 3 D   Figure 4: Meshed Model in 3 D

2.3.2. Analysis of crack propagation

Ansys software is implementedtoanalyse the crack propagationof cracked gear. Itis afinite element software for static and dynamic analysis for two-dimensional and three-dimensional problemsof the plane strain and plane strain. The gear model was imported in Ansys software from SOLIDWORKS. Ansys software generated the meshing in the gear model. A meshed model of the gear sector is shown in Figure 4. The geometric andmaterial constraints of the gear aresimilar to the test gear, used in the experimental setup.The required geometrical and material properties of SAE 8620 were assigned to the gear model. The physical and geometrical properties were referred from Tables 1,3..The model is having the six-node tetrahedron element. The meshed model had 62936 nodes and 10364 six-node tetrahedron elements. Ansys had calculated stress intensity factor (SIF), crack increment and the crack propagation angle. SIF was calculated for the crack increments associated with the number of cycles.

The Paris law in Eq. (14) was used in numerical simulation to estimate the crack propagation rate. Next, The service life is estimated by the functional relationofcrack propagation rate and SIF.Paris law constants C and m were also assigned. The values C is 2.2 x 10^{-12} mm/(cycle.MPa.mm$^{0.5}$) and m is 2.7.The tangential tooth load was assumed to be acting at HPSTC.The effect of radial component is not considered as it is assumed to be negligible. For the required boundary condition, the left sides of the gear sector and hub nodes were fixed as shown in Fig 5.

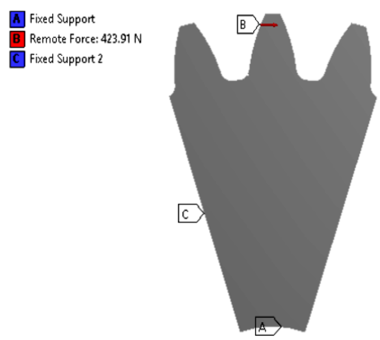


Figure 5: Boundary condition

The boundary condition in which, the tangential force of 423.91 N pulsating in nature, is applied at the HPSTC, as shownin Figure 5.The fixed supports are applied on the left side and at the shaft hole position of the gear sector.

3. Results and Discussion

3.1. Numerical simulation

Numerical simulation was carried out for determination of stress intensity parameter, fatigue crack propagation life and crack trajectory. The terms are explained and determined below.

3.1.1. Mode-I Stress Intensity factor

The most dominating factor in crack propagation analysis is the mode-I SIF (K_I) which is required to determine the service life as well as crack propagation direction.It canalso establish functional relationship between $\boldsymbol{\Delta K} = \boldsymbol{f}(\boldsymbol{a})$.

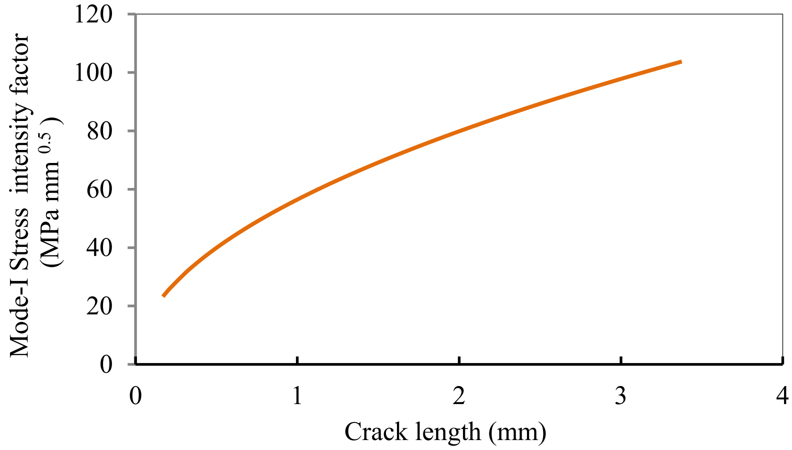


Figure 6: Mode1 SIF versus crack length

The functional relation between the Mode-I SIF and crack extension *a*, where *K* is determined numerically.Figure 6shows a graph of the no. of load cycles and crack extension, which was found from the simulation of crack propagation on gear. The numerical value of K_I is higher than K_{II} (K_I is 95% of K, so K_{II} is ignored from calculation), so the Mode-I facture toughness, K_{IC} is considered as critical parameter. The critical crack length can be determined from $K_{IC}$with applied stress of 28.44 N/mm^2.

3.1.2. Service life estimation by Numerical simulation

Numerical simulation was performed using Ansys software as explained in section 2.2. The numbers of cycles with respect to crack length were determined and it is shown in Figure7. In observation, the rise in crack length is very less up to mid-point of the curve,andcrack length increases rapidly in further cycles.

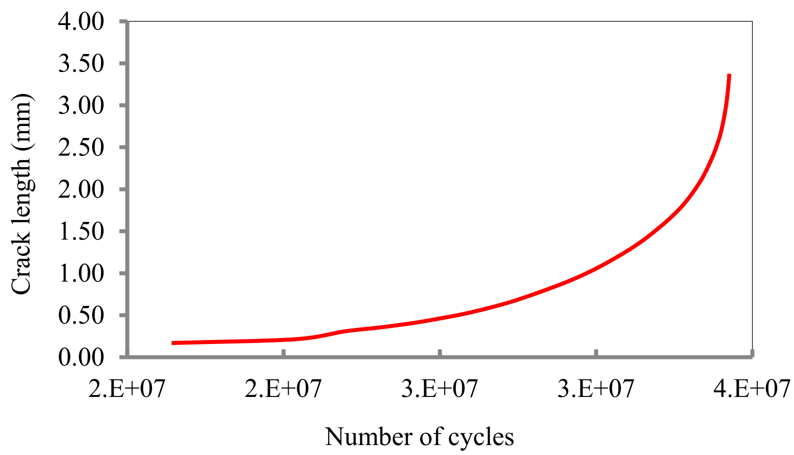


Figure 7: A graph of crack length and no. of cycles

The numerical simulation is performed up to the crack length of 3.3759 mm for number 3.4261×10^7 cycles. But the final crack length just before gear tooth failure is assumed to be 4.2 mm So, it is necessary to find the no. of cycles up to crack length of 4.2 mm (i.e. final crack length).

The crack lengths (next to 3.3759 to 4.20 mm) got from numerical simulation) corresponds to no. of cycles are obtained byI_p theory Eq.(10) are shown in Table 3 and used in Graph shown in Figure 7.

Table 3: Crack lengths and no. of cycles

Crack length	3.65	3.74	3.83	3.92	4.01	4.12	4.20
No. of cycles	34278000	34295000	34312000	34329000	34346000	34363000	34380000

3.1.3. The useful service life of the gearusing I_p Theory

It is required to find the numbers of cycles from initial to final crack length by Eqs. (10). The crack length just before the tooth failure in experimental method is 4.2 mm. The initial length of crack, ai is 0.125 mm and final crack length, a_f is 4.2 which was determined from the experiment. The parameters C is 7.002 x 10^{-9}mm/[cycle (MPa)^2) and m=0.5678 The values were obtained from the fatigue crack growth experiment on single edged notched specimen (SENB).The C and m parameters are determined similarly to Paris law using SENB specimen. Final number of cycles,

$$N = \int_{a_i=0.125}^{a_f=4.2} \frac{da}{\left(7.002\,\mathrm{x}\,10^{-9}\left(1.12\,\mathbf{x}\,28.44\,\mathbf{x}\sqrt{\boldsymbol{\pi a}}\right)^{0.5678}\right)}$$

$$= 3.5145 \times 10^7 \, cycles$$

Hence, theuseful service life of the gear is **3.5145 x 10^7**cyclesup to final crack length of 4.2 mm.

The final crack length using numerical simulation was 3.3759 mm only. But the final crack length was assumed to be 4.2mm. The remaining crack lengths corresponds to no. of cycles are obtained by Eq.(10) of I_p theory and used in Graph shown in Figure 7.

The servicelifecycle of the gear bysimulation method with I_p theory is3.5145 x 10^7 cycles.

3.2. Determination of crack trajectory using numerical method

3.2.1. Crack trajectory using numerical simulation with Ip theory

For identification of the crack trajectorypath in mixed modes-I and II, a spur gear with an preliminary crack under tangential tooth load P_t was used as shown in Figure 5.The crack path data obtained from numerical simulationwasused for determination of crack trajectory. The numerical simulation with the 'I_P' theory was used to find crack direction in the spur gear tooth.Figure 8 shows the crack trajectory at the gear tooth.

SIF at crack-tip of gear tooth

$$K = \sigma_{max}\sqrt{(\pi . a)} \tag{17}$$

Mode-I SIF,

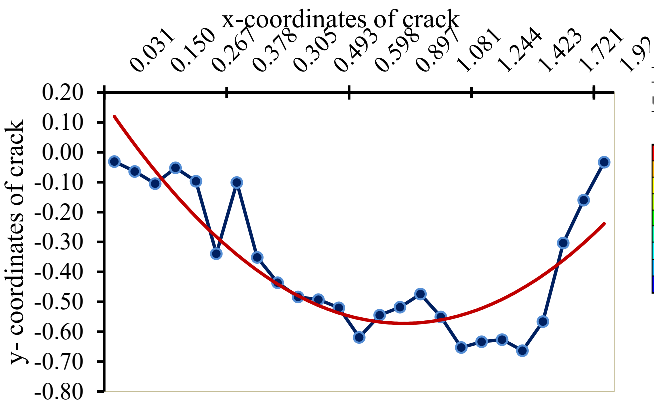
$$K_I = K \sin^2 \theta \tag{18}$$

Mode-II SIF,

$$K_{II} = K \sin(\theta) . \cos(\theta) \tag{19}$$

Eqs.(1) to (8) give an angleθat every crack advancement. Thus, angles are found out by the 'I_P' theory for the observations and the graph was plotted. Using crack data, the path of the crack was determined shown in Fig.15. The coordinates x and y of the crack tip were considered for finding the crack path crack trajectory. The fatigue crack propagation angles were determined from Eqs. (1) to (8).

The procedure to determine the crack trajectory: 1) First crack length with respect to no of cycles was obtained from numerical simulation. 2) Stress Intensity factor was determined 3) Initial the crack length of 0.125 mm and angle of crack of 45°were considered.For the next crack length was assumed to be inclined at 0°.The respective stress intensity factor K, the mode-I and mode-II SIF are calculated from the Eqs.(18) and (19). Putting the values in Eq.(7), the crack propagation angle θwasobtained for every crack advancement. The procedure is repeated for remaining cycles. Thus crack trajectory is determined. Figure 9 shows the output of Ansys software i.e. the crack trajectory at the root of the gear. The crack trajectory is in concave nature and along the tooth thickness.



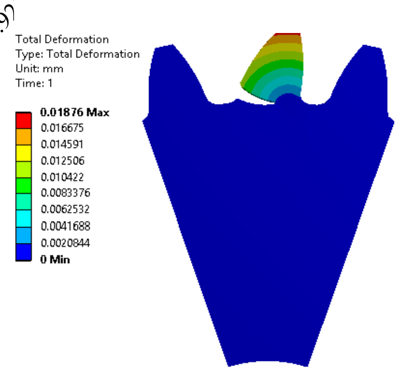


Figure 8: The crack trajectory at the root of the gearFigure8. The crack path using FEA

It is observed from Figure.8 and Figure 9 that the crack trajectory is curved and it is propagated from crack initiation pointtowards the opposite side of the tooth root.

4. Conclusion

Numerical simulation conducted for the service life assessment and thecrack trajectory at the tooth root,due to bending fatigue. The gear of tumbler gear mechanism is considered for this study to predict the stage before the gear tooth failure.

- $I_P$theory has been successfully implemented to estimate the service life of the spur gear.Estimated life cyclesby$I_P$theoryare **3.5145 x 10^7**.
- The crack trajectoriesof the gear specimen in FEA and FEA with I_P theory are found in close match. The curved crack path profilesarekeenly observedand found that, it propagatestowardsthe opposite side of the gear tooth along the face width.

In the present work, the simulation gives the perfect idea about prediction of the spur gear service life and itsearly maintenance or replacement before the failure.

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